

Math Strategies and Interventions for Elementary Students

**Created For
Shiawassee Regional Education Service District (Corunna, MI)**

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2009-2010**

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MATH INTERVENTION SKILL SEQUENCE BY GRADE

<i>Grade</i>	<i>Skill</i>	<i>Intervention Type</i>
1 st	Addition/Sums to 6	Flash Cards
	Addition/Sums to 12	Flash Cards
	Subtraction 0 to 5	Flash Cards
	Subtraction 0 to 9	Flash Cards
	Fact Families Addition/Subtraction 0 to 9	Practice Set – Same as Skill
2 nd	Addition Facts 0 to 20	Flash Cards
	Subtraction Facts 0 to 9	Flash Cards
	Subtraction Facts 0 to 12	Flash Cards
	Subtraction Facts 0 to 15	Flash Cards
	Subtraction Facts 0 to 20	Flash Cards
	Mixed Subtraction/Addition 0 to 20	Flash Cards
	Fact Families Addition/Subtraction 0 to 20	Practice Set – Same as Skill
	2-Digit Addition Without Regrouping	Practice Set – Same as Skill
	2-Digit Addition With Regrouping	Practice Set – Same as Skill
	2-Digit Subtraction Without Regrouping	Practice Set – Same as Skill
	2-Digit Subtraction With Regrouping	Practice Set – Same as Skill
	3-Digit Addition W/ and W/O Regrouping	Practice Set – Same as Skill
	3-Digit Subtraction W/ and W/O Regrouping	Practice Set – Same as Skill
	2 nd Grade Monthly Math Probe	Practice Set – Same as Skill
3 rd	Addition/Subtraction Facts 0 to 20	Flash Cards
	Fact Families with Addition/Subtraction 0 to 20	Practice Set – Same as Skill
	3-Digit Addition W/ and W/O Regrouping	Practice Set – Same as Skill
	3-Digit Subtraction W/ and W/O Regrouping	Practice Set – Same as Skill
	2- and 3-Digit Addition/Subtraction W/ and W/O Regrouping	Practice Set – Same as Skill
	Multiplication Facts 0 to 9	Flash Cards
	Division Facts 0 to 9	Flash Cards
	Fact Families Multiplication and Division 0 to 9	Practice Set – Same as Skill
	Add/Subtract Fraction With Like Denominators (3rds, 4ths, 8ths, and 10ths, no grouping)	Practice Set – Same as Skill
	Single Digit Multiplied by Double/Triple Digit W/ and W/O Regrouping	Practice Set – Same as Skill
	Single Digit Divided into Double/Triple Digit Without Remainders	Practice Set – Same as Skill
	Add/Subtract Decimals to the Hundredths	Practice Set – Same as Skill

MATH INTERVENTION SKILL SEQUENCE BY GRADE CONTINUED

<i>Grade</i>	<i>Skill</i>	<i>Intervention Type</i>
4 th	Multiplication Facts 0 to 12	Flash Cards
	Division Facts 0 to 12	Flash Cards
	Fact Families Multiplication/Division 0 to 12	Practice Set – Same as Skill
	Single Digit Multiplied by Double Digit W/ and W/O Regrouping	Practice Set – Same as Skill
	Double Digit Multiplied by Double Digit W/ and W/O Regrouping	Practice Set – Same as Skill
	Single Digit Divisor into Double Digit Dividend W/ and W/O Remainders	Practice Set – Same as Skill
	Single and Double Digit Divisor into Single and Double Digit Dividend with Remainders	Practice Set – Same as Skill
4 th	Add/Subtract Fractions With Like Denominators and With No Regrouping	Practice Set – Same as Skill
	Multiply Multi-Digit Numbers By Two Numbers	Practice Set – Same as Skill
	Add/Subtract Decimals to the Hundredths	Practice Set – Same as Skill
5 th	Multiplication Facts 0 to 12	Flash Cards
	Division Facts 0 to 12	Flash Cards
	Fact Families Multiplication/Division 0 to 12	Practice Set – Same as Skill
	Multiply 2 and 3 Digit W/ and W/O Regrouping	Practice Set – Same as Skill
	Single Digit Divisor Divided Into Double Digit Dividend With Remainders	Practice Set – Same as Skill
	Single Digit Divisor Divided Into Double and Triple Digit Dividend With Remainders	Practice Set – Same as Skill
5 th	Reduce Fractions to Simplest Form	Practice Set – Same as Skill
	Add/Subtract Proper Fractions/Mixed Numbers With Like Denominators With Regrouping	Practice Set – Same as Skill
	Add/Subtract Decimals	Practice Set – Same as Skill
	Multiply/Divide Decimals	Practice Set – Same as Skill
	Double Digit Divisor Into 4 Digit Dividend	Practice Set – Same as Skill
	Multiply and Divide Proper and Improper Fractions	Practice Set – Same as Skill

Sources:
VanDerHeyden, A. <http://www.gosbr.net/>

SEQUENCE FOR TEACHING FRACTIONAL CONCEPTS

PURPOSE: Below are suggestions for the progression in working with students on **fractions** and understanding **concepts behind fractions**. Intended for Elementary and Middle school.

MATERIALS:

- Fractional models and manipulatives
- Graph paper

PROCEDURE

For the student

1. Manipulatives and concrete models
 - Example: Manipulating fractional blocks and pegs
2. Matches fractional models
 - Example: Matching halves, thirds, and fourths
3. Points to fractional model when name is stated by another
 - For example: The teacher says “half” and the student selects a model of “half” from several distracters
4. Names fractional units when selected by another
5. Draws diagrams or uses manipulatives to represent fractional units
6. Writes fraction names when given fractional drawings
7. Uses fractions to solve problems

Sources:

Bos, C. S., & Vaugh, S. (2002). *Strategies for teaching students with learning and behavior problems* (5th ed.). Boston, MA: Allyn and Bacon.

VanDerHeyden, A., & Witt, J. <http://www.gosbr.net/>

STAR: A NUMBER WRITING STRATEGY

PURPOSE: Strategy used to assist students in **recalling, reciting, and writing numerals.**

MATERIALS:

- Pretest
- Posttest

PROCEDURE:

1. Pretest and Obtain Commitment to Learn Strategy

- Administer pretest to determine student's baseline number writing skills
- Pretest can consist of having student write numbers 0 through 9
- After student writes numbers, the teacher shows a chart of correctly formed numerals and brings the differences to the student's attention
- Show numbers as prices to explain how unacceptable it is to make the numbers incorrectly
- Have the student make a commitment to learning a strategy for writing numbers

2. Describe the Strategy

- Formation of numbers is taught first using multisensory experiences
- Teacher demonstrates formation of a number while reciting its associated saying. Sayings for numbers are as follows:
 - 0: The woman went around in a circle until she got home
 - 1: The man went straight down, like a stick
 - 2: The woman went right and around, slide down the hill to the left, then made a line across the ground
 - 3: The man went right and around, then around again
 - 4: The woman went down the street, turned to the right, then back to the top for a straight ride down
 - 5: The man went down the street, around the corner, and his hat blew off
 - 6: The woman made a curve and then a circle at the bottom
 - 7: The man made a line across the top, then slide down the hill to the left
 - 8: The woman made a half circle to the left, another to the right, and then she found her way back up to the top again
 - 9: The man made a small circle then a straight line down
- Child repeats the saying while forming the number:
 - In the air with large motor movements
 - Using sand, clay, sandpaper, crayon on paper held over a screen, and a marker
 - Pencil and paper
- Teach STAR strategy for when student is unable to recall correct formation
 - S = STOP and ask myself what I am expected to do (e.g. write the #)
 - T = THINK of using a saying to help in forming the number
 - A = ASK myself which saying should be used for this number
 - R = RECITE the saying while I write the number
- Teacher should point out various times to use the STAR strategy and demonstrate each step
- A STAR Strategy Card can be made with the steps of the strategy on 1 side and textured numerals with the saying on the other side

3. Model the Strategy

- Demonstrate how to use the strategy by talking through the steps, reciting the saying aloud, and writing the number on the board for the student to see
- Allow student to practice in self-verbalization by asking the student "what should I do next?" when modeling the strategy

4. Memorization of the Strategy

- Students can memorize the sayings and the STAR strategy via verbal rehearsal
- The STAR Strategy Card can be used as a cue during rehearsal

5. Practice with Controlled Materials

- Practice 1 number at a time: dictate, simple math, or answer a question
- Provide feedback on elements of the strategy that are being done correctly
- Provide corrective feedback to improve performance

6. Practice with Grade-Appropriate Tasks

- Student should apply strategy to classroom materials
- Provide frequent opportunities for practice. For example:
 - Writing multi-digit numbers
 - Number pages or lines
 - Complete math problems
 - Write the date
 - Pretending to operate a store

7. Administration of Posttest

- Require student to write numbers and compare to pretest
- Show student results and explain that they need to remember to use the strategy whenever they need to write a number

8. Generalization

- Monitor use of strategy in other situations
- Ask children to report when they use the strategy outside the classroom
- Review the steps periodically to encourage generalization

Sources:

Boom, S. E., & Fine, E. (1995). Star – A number writing strategy. *Teaching Exceptional Children*, 42-45.

RULES TO LOWER THE AMOUNT OF MEMORIZATION IN MATH

PURPOSE: Strategy used to teach students **general math rules**. Relationships and rules are taught to assist in figuring out challenging problems. Students should be taught to recognize and use math rules and relationships along with concepts being taught.

MATERIALS:

- Manipulatives to demonstrate concrete examples of the rules (*optional*)

PROCEDURE:

- 1. Explicitly teach students each of the following steps in the strategy

ADDITION:

- Any number + 0 is the original number
- Any number + 1 is the next larger number
- The order of numbers in an addition problem doesn't change the answer

SUBTRACTION:

- Any number take away 0 is the original number
- Any number take away the same number is 0
- Any number take away 1 is the next smaller number
- In subtraction, when the bottom number in the ones column is bigger than the top number in the ones column, the ten is traded

MULTIPLICATION:

- Any number times 0 equals 0
- Any number times 1 equals the original number
- 2 times any number equals the number added to itself
- Changing the order of the numbers in multiplication does not change the answer

DIVISION:

- 0 divided by any number equals 0
- Any number divided by 1 equals the number
- Any number divided by the same number equals 1

Sources:

Miller, S. P., Strawser, S., & Mercer, C. D. (1996). Promoting strategic math performance among students with learning disabilities. *LD Forum*, 21, 34-40.

Reid B., & Lienemann, T. <http://www.unl.edu/cis/index.shtml>

SELF-MONITORING STRATEGIES IN ARITHMETIC

PURPOSE: Strategy to help students who have learned single-digit arithmetic, but struggle with **multi-step arithmetic problems**. This strategy reminds students of the steps involved in computing answers to arithmetic problems. It teaches a self-monitoring procedure and mnemonic devices to help students remember how to solve problems once the self-monitoring procedure has been faded.

MATERIALS:

- Addition, subtraction, or multiplication worksheets, depending on the arithmetic strategy that is implemented

PROCEDURE:

1. Provide students with a checklist that prompts them to remember how to complete an arithmetic task. The checklist should include:
 - A careful analysis of the skill taught and identification of the critical steps to be performed
 - Write down the steps at a level that can be understood by the student
 - Organize the written statements in sequential order
 - Place the checklist next to each problem as it is done
2. Explain checklist and give concrete examples of how to use it. Once the students are able to use the procedure without assistance, the teacher can discuss a mnemonic strategy for remembering the steps in the checklist
3. When the students are showing success with the checklist, encourage them to use the mnemonic device and wean them off of the checklist. First remove the steps from the top of the worksheet then omit the checklist adjacent to individual problems
4. If transitions are difficult, the checklist can be placed on a note card or a chart on the wall

ADDITION PROBLEMS

- Critical steps:
 - Start in the 1's column
 - Add together the numerals in each column
 - Determine whether regrouping is necessary
 - Check to see whether the correct numeral has been carried to the next column where regrouping is necessary
- **SASH** can be used as a mnemonic strategy to help students remember the steps
 - **S**tart in the 1's column
 - **A**dd together the numerals together
 - **S**hould I carry a numeral?
 - **H**ave I carried the correct numeral?

SUBTRACTION PROBLEMS

- Critical steps:
 - Start in the 1's column
 - Determine which numeral in each column is bigger
 - Regrouping if the bottom numeral is bigger
 - Check basic facts
- **4Bs** can help students remember the steps
 - **B**egin? In the 1s column

- **Bigger?** Which number is bigger?
- **Borrow?** If bottom number is bigger I must borrow
- **Basic Facts?** Remember them. Use Touch Math if needed

MULTIPLICATION PROBLEMS

- Critical steps:
 - Multiply the 1s column
 - Carry any 10s over to the 10s column
 - Multiply the bottom 1s digit with the top 10s digit
 - Add any number that was carried in step 2 to the product of step 3
- **MAMA** can be used as a mnemonic strategy to help students remember the steps
 - **M**ultiply the 1s column
 - **A**cross to the 10s
 - **M**ultiply the bottom 1s digit with the top 10s digit
 - **A**dd any number that was carried in step 2

Sources:

Frank, A. R., Brown, D. (1992, Winter). Self-monitoring strategies in arithmetic. *Teaching Exceptional Children*, 52-53.
Reid B., & Lienemann, T. <http://www.unl.edu/cis/index.shtml>

LAMPS & SLOBS

PURPOSE: Strategies to help students remember the **regrouping process of borrowing** and **carrying** in addition and subtraction.

MATERIALS:

- Addition or subtraction worksheets, depending on the arithmetic strategy implemented

PROCEDURE:

1. Explicitly teach students the steps of the strategies by providing examples

ADDITION – LAMPS

- **Line**
 - Line up the numbers – This is important with extensive columns and with numbers with decimal points
- **Add**
 - Add the right column of numbers and ask yourself ...
- **More**
 - More than nine? If so, do more steps
- **Put**
 - Put the ones below the column
- **Send**
 - Send the tens to the top of the next column

Subtract

ADDITION EXAMPLE

$$\begin{array}{r}
 62.1 \\
 42. \\
 7.1 \\
 + 4.4 \\
 \hline
 \end{array}$$

- The numbers have been lined in columns according to their decimal points
- In adding the right column the sum is 13 and more than 9
- Put the ones (3) below the column and send the tens (1) to the top of the next column
- Repeat from “Add” for each of the next columns

SUBTRACTION – SLOBS

- **Smaller**
 - Smaller – Follow Steps
- **Larger**
 - Larger – Leap to subtract
- **Off**
 - Cross off the number in the next column
- **Borrow**
 - Borrow by taking 1 ten and adding to the next column
- **Subtract**
 - Subtract

SUBTRACTION EXAMPLE

72

-46

- Look at the top number on right (2) and see if it is smaller than the lower number (6)
- If it is larger, student will leap to subtract. If it is smaller (which it is in this example), the student must follow the steps.
- Cross off the number in the next column (7)
- Borrow 1 ten from that column by reducing the number by 1 and adding 10 to the other number (which makes 2 now 12)
- Subtract 6 from 12
- Repeat steps if there are more digits to be subtracted

6. Daily, or several times a week, provide students with sets of training sheets and have them follow the Cover, Copy, and Compare procedure

7. Once or twice a week, administer the assessment sheets that correspond with the training sheets

8. When students reach mastery on one set of problems, provide them with another set

- Mastery is defined as 90% or better accuracy and/or 40 digits correct per minute

Sources:

Reetz, L., & Rasmussen, T. (1998). Arithmetic mind joggers. *Academic Therapy*, 24, 79-82.

Reid B., & Lienemann, T. <http://www.unl.edu/cis/index.shtml>

MATH COMPUTATION: INCREASE ACCURACY BY INTERMIXING EASY AND CHALLENGING PROBLEMS

PURPOSE: Improve **accuracy** and **positively influence the attitude** of students when completing math-fact worksheets.

MATERIALS:

- Math computation worksheets with a mixture of difficult and easy problems
- Answer keys

PROCEDURE:

1. Teacher identifies 1 or more ‘challenging’ problem types that are matched to the student’s current math-computation abilities

2. Teacher identifies ‘easy’ problem types that the student can complete quickly

3. Teacher creates math computation worksheets with ‘easy’ problems interspersed at a fixed rate among the ‘challenging’ problems

4. To create worksheets:

- Seatwork/independent work: ‘Easy’ and ‘challenging’ problems should be interspersed at a 1:1 ratio (every ‘challenging’ problem is followed by an ‘easy’ problem)
- Student reads problems aloud and asked to solve problems mentally and write down the answer: Items should be at a ratio of 3:1 (every 3 ‘challenging’ problems are followed by 1 ‘easy’ problem)
- www.interventioncentral.org can be used to create worksheets for free

Sources:

Hawkins, J., Skinner, C. H., & Oliver, R. (2005). The effects of task demands and additive interspersal ratios on fifth-grade students’ mathematics accuracy. *School Psychology Review, 34*, 543-555.

Wright, J. www.interventioncentral.org

MULTIPLICATION ATTACK STRATEGY

PURPOSE: Strategy used to help teach **multiplication facts**. It is important that the teacher determines whether the student possesses the necessary basic skills before this strategy is taught.

MATERIALS:

- Multiplication worksheets

PROCEDURE:

1. Assess students' basic skills prior to teaching the strategy. Prerequisite skills for this strategy are:

- Say numbers 0 to 100
- Write numbers 0 to 100
- Name x (times) and = (equal) signs
- Make the number of marks indicated by numerals 0 to 10
- Count by numbers 1 to 10
- End counting-by sequences in various positions
- Coordinate counting-by and touching-marks actions

2. Explicitly teach students each of the following steps in the strategy

3. Read the problem

- Example: $2 \times 5 = \underline{\hspace{2cm}}$

4. Point to a number that you can count by

- Example: Student points to the number 2

5. Make the number of marks indicated by the other number

- Example: Since the other number is 5, the student would make 5 marks like this: / / / / /

6. Begin counting by the number you know how to count by and count up once for each mark, touching each mark

- Example: "2, 4, 6, 8, 10"

7. Stop counting when you have touched the last mark

8. Write the last number you said in the answer space

- Example: $2 \times 5 = \underline{10}$

Sources:

Cullinan, D., Lloyd, J., & Epstein, M. H. (1981). Strategy training: A structured approach to arithmetic instruction. *Exceptional Education Quarterly*, 2, 41-49.

Reid B., & Lienemann, T. <http://www.unl.edu/cis/index.shtml>

IMPROVING MATH COMPLETION RATES AND ACCURACY WITH FREE TIME

PURPOSE: Improve **accuracy** and **completion rates** of math class work with a group-oriented free-time contingency

MATERIALS:

- Math worksheets

PROCEDURE:

1. Tell students they will be able to earn free time if the class correctly completes a specified average number of problems during each math class work session
2. Set free-time period from 5 to 15 minutes, depending on length of math period
3. Select criterion for correct assignment completion (ex. 70% correct)
4. Set a fixed amount of time for class work completion during the math instructional period
5. At the end of the fixed time, have students exchange papers to check and report number of problems completed correctly (without names) to you
6. If class average meets criterion, praise students for hard work and award free time
7. If class average does not meet criterion, encourage students to try harder next time and continue with math lesson
8. When class has met criterion for 5 consecutive days, increase it by several more percentage points
9. Continue to increase gradually until students are performing at 90% or better accuracy rate

Sources:

- Johnston, R. J., & McLaughlin, T. F. (1982). The effects of free time on assignment completion and accuracy in arithmetic: A case study. *Education and Treatment of Children*, 5, 33-40.
- Rathvon, N. (1999). *Effective school interventions: Strategies for enhancing academic achievement and social competence*. New York, NY: The Guilford Press.

TEACHING MATH FACTS

PURPOSE: Improve **accuracy** and **fluency** in **addition, subtraction, multiplication, or division**. Requires approximately 7 minutes per day.

MATERIALS:

- 5 math probes (i.e., worksheets containing math problems for the problem skill)
- Timer
- Monitoring chart
- Draw a line under the first 2-3 rows of problems on the worksheet
- Review progress and change materials weekly
- Ask students to select 3 items/activities from the “Things I Would Like to Earn” list

PROCEDURE:

1. Instruct students to get out materials
2. Students write name and date on math sheet
3. Work all the problems above the line on the worksheet with teacher’s help
4. Set timer for 2 minutes
 - Cover the practice problems above the line
5. Work problems below the practice line for 2 minutes
 - When timer rings, stop working
6. Score paper
 - Count number of problems correct
 - Write score at top of math sheet
7. Write score on progress monitoring chart
 - Did the student beat their score?
 - Circle yes or no on progress monitoring chart
 - If student beat their score, allow a reward to be chosen from the reward menu

Sources:
VanDerHeyden, A., & Witt, J. <http://www.gosbr.net>

COVER, COPY, AND COMPARE: INCREASING MATH FLUENCY

PURPOSE: Improve **accuracy** and **speed** with basic math facts.

MATERIALS:

- Worksheets (training) with 10 math problems listed down the left side of paper
- Answers provided for each problem, one per student, 1-3 sets per session
- Worksheets (assessment) with 10 math problems listed down left side of paper with blanks next to each problem for written responses
- 3" X 5" index cards, 1 for each student

PROCEDURE:

1. Tell students they will be learning a new method of improving math performance called Cover, Copy, and Compare
2. Give training sheets to students and use overhead projector to display a training sheet for introductory session (if desired)
3. Conduct training session to teach students to follow Cover, Copy, and Compare procedure
4. Cover, Copy, Compare procedure is as follows:
 - Silent read first problem and the answer on the left side of paper
 - Cover that problem and answer with an index card
 - Write the problem and answer from memory on the right side of the page
 - Uncover the problem and answer on the left side to check written response
 - Evaluate response
 - If problem and answer are written incorrectly, repeat procedure with that item before moving to next item
 - Repeat procedure with rest of problems on worksheet
 -
5. After demonstrating steps, have students complete 1 or more training sheets and provide corrective feedback is needed
6. Daily, or several times a week, provide students with sets of training sheets and have them follow the Cover, Copy, and Compare procedure
7. Once or twice a week, administer the assessment sheets that correspond with the training sheets
8. When students reach mastery on one set of problems, provide them with another set
 - Mastery is defined as 90% or better accuracy and/or 40 digits correct per minute

Sources:

- Lee, M. J., & Tingstrom, D. H. (1994). A group math intervention: The modification of cover, copy, and compare for group application. *Psychology in the schools, 31*, 133-145.
- Rathvon, N. (1999). *Effective school interventions: Strategies for enhancing academic achievement and social competence*. New York, NY: The Guilford Press.

MATH COMPUTATION: INCREASE ACCURACY AND PRODUCTIVITY RATES VIA SELF-MONITORING AND PERFORMANCE FEEDBACK

PURPOSE: Improve **accuracy** and **fluency** on math computation worksheets by self-monitoring their speed by charting their daily progress and earning rewards for improved performance.

MATERIALS:

- Math computation worksheets
- Answer keys
- Student self-monitoring charts

PROCEDURE:

1. Preparation for intervention

- Teacher selects 1 or more computation problem types that the student needs to practice
- Teacher prepared progress monitoring chart (vertical axis extends from '0' to '100' and horizontal axis is labeled 'date' or see <http://www.interventioncentral.org>)
- Teacher creates reward menu that the student can choose from

2. Student is given a pre-selected amount of time (example, 5 minutes) to complete as many problems on the computation worksheet as possible

3. Student uses answer key to check their work, giving credit for each correct digit in an answer

4. Student plots their computational fluency score on the progress monitoring chart and writes the current date at the bottom of the chart below the data point

5. Allow student to choose from the reward menu if they exceed their most recent fluency score

Sources:

Bennett, K., & Cavanaugh, R. A. (1998). Effects of immediate self-correction, delayed self-correction, and no correction on the acquisition and maintenance of multiplication facts by a fourth-grade student with learning disabilities. *Journal of Applied Behavior Analysis, 31*, 303-306.

Shimabukuro, S. M., Prater, M. A., Jenkins, A., & Edelen-Smith, P. (1999). The effects of self-monitoring academic performance on students with learning disabilities and ADD/ADHD. *Education and Treatment of Children, 22*, 397-414.

Wright, J. www.interventioncentral.org

MULTIPLICATION WHEEL

PURPOSE: Improve **accuracy** and **fluency** with basic math facts by teaching students the visual patterns that emerge from completing the Multiplication Wheel.

MATERIALS:

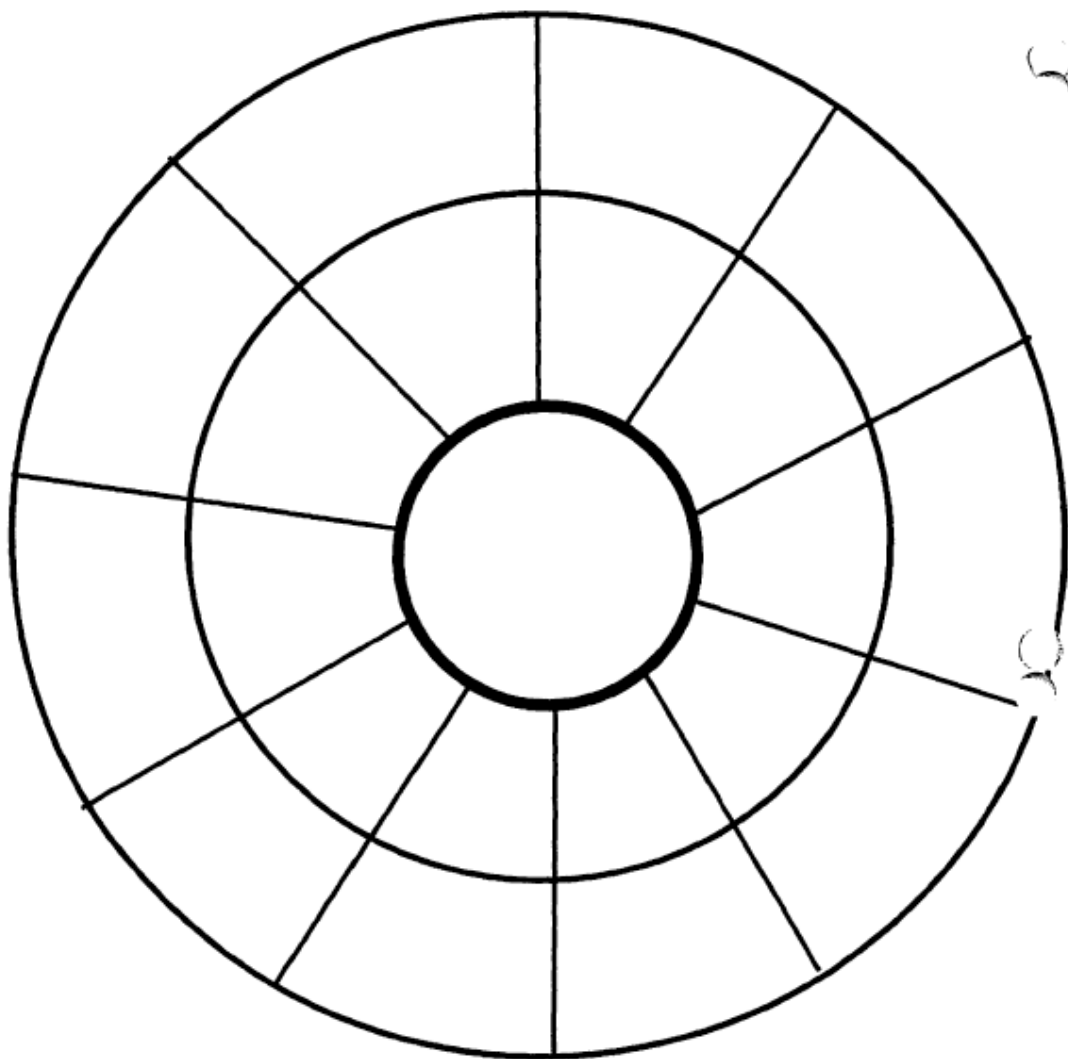
- Copy of the Multiplication Wheel (see attached)
- Sets of math worksheets with multiplication problems

PROCEDURE:

1. Have student(s) fill in the multiples of the number 2 on a blank Multiplication Wheel
2. Discuss any visual patterns that emerge
 - Example: To get the next box up, all you do is bring the second number up and add the central number to the first number
3. Have the student(s) complete the multiplication circles for the numbers 3 through 9
4. Have the student complete a multiplication fact sheet using the wheel as a reference if needed
5. Have the student check his/her work using the wheels
6. Give the student(s) the same fact sheet to complete without the use of the wheels
 - Allow only 3 minutes to complete this sheet
7. Check to see how many are correct
 - Express the number of items correct as a fraction over the total possible, including any items omitted due to time

Sources:

Ingham Intermediate School District (Mason, MI) '05-'06 Inservices: *Accountability in the 21st Century, Supporting All Learners Through Collaboration, and Using Your Toolbox to Create a Well-Oiled Machine.*



MULTIPLICATION GRID

PURPOSE: Improve **accuracy** and **fluency** with basic math facts by filling in Multiplication Grid (10 X 10) each day and by providing a tool for students to use when completing multiplication facts.

MATERIALS:

- Multiplication Grid (see attached)
- Sets of math worksheets with multiplication problems

PROCEDURE:

1. At the beginning of **every** math seatwork period, have students complete a 10 X 10 grid of products
 - Allow students to use any method to complete the sheet, including counting on and off from adjacent products, identifying and applying patterns, or even using straight edges down from the top margin and across the left and counting the enclosed squares
 - These short cuts help with the learning
2. After students complete the grid, have them date it and allow them to use it for that one day
3. After that math seatwork period ends, students do not use the same grid for the next seatwork period.
4. Repeat this procedure for the next math seatwork period
 - Even if some products are never memorized, the student at least has a tool for deriving them when they are needed
5. Send grid home for homework occasionally

Sources:

Ingham Intermediate School District (Mason, MI) '05-'06 Inservices: *Accountability in the 21st Century, Supporting All Learners Through Collaboration, and Using Your Toolbox to Create a Well-Oiled Machine.*

Brenner, R. (2001).

Name _____ Date: _____

X	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

From Ron Benner 5-1-2001 (eclectic data from the past)

IMPROVING MATH PERFORMANCE WITH EXPLICIT TIMING

PURPOSE: Improve **fluency** with basic math facts by timing math performance.

MATERIALS:

- Stopwatch
- Kitchen timer with bell
- Set of math worksheets with 100 basic problems on one side and sheets stapled together (addition, subtraction, etc.)

PROCEDURE:

1. At the beginning of the math seatwork period, tell students that the work period is 30 minutes and that you will be timing the period as a way of helping them improve their performance
2. Tell students you will set the timer for the amount of time in the period and that you will also be timing them in 1-minute increments
3. At the beginning of each timing, say “Pencils up, ready, begin!” to signal to students to begin
4. At the end of the 1-minute interval, say “Stop!” and have students draw a line after the last problem answered
5. Repeat this procedure throughout the 30 minute period until the last timing is completed
6. When timer rings, announce that the work period is over
7. Teach students to stop when the timer rings, even if they are in the middle of a 1-minute timed period

Sources:

- Rathvon, N. (1999). *Effective school interventions: Strategies for enhancing academic achievement and social competence*. New York, NY: The Guilford Press.
- Van Houten, R. & Thompson, C. (1976). The effects of explicit timing on math performance. *Journal of Applied Behavior Analysis*, 9, 227-230.

MATH REVIEW: PROMOTE MASTERY OF MATH FACTS THROUGH INCREMENTAL REHEARSAL

PURPOSE: Improve **fluency** with basic math facts.

MATERIALS:

- Index cards
- Pen

PROCEDURE:

1. Preparation for intervention

- Tutor writes math facts student is expected to master but without the answer on the index cards
- Tutor reviews collection of math fact cards with student
- If student orally answers within 2 seconds, put in 'known' pile
- If student does not orally answer within 2 seconds, put in 'unknown pile'
- Tutor randomly selects 9 cards from pile of known math facts and discards rest of 'known' deck

2. Tutor takes 1 card from the 'unknown' deck and reads math fact aloud, provides answer, and asks the student to read and answer the same problem

3. Tutor takes 1 card from the 'known' deck and pairs it with the unknown problem

- When shown the 2 cards in sequence, student is asked to read off the problem and answer
- Student is judged to be successful on a problem if they provide correct answer within 2 seconds
- If student is incorrect or takes longer than 2 seconds, tutor reads math fact and gives answer
- Student reads the same math fact and provides answer
- Continue to do this until all cards are answered correctly within 2 seconds

4. Tutor takes another problem from the 'known' deck and adds to the collection of math facts being reviewed ('review' deck)

- Tutor prompts student to read and answer whole series of math facts in 'review' deck, beginning with the unknown fact and then moving through growing series of 'known' facts that follow it

5. When 'review' deck has expanded to include 1 unknown fact followed by 9 known facts, the last known fact is discarded

- The previously unknown fact that the student just successfully practiced is now treated as a known fact and is included as the first item in the 9 card 'known' facts deck for future drills

6. Present student with new math fact to answer taken from the 'unknown' deck

- With each 'unknown' fact the sequence is repeated as described above until 'unknown' fact is grouped with the 'known' facts

Sources:

Burns, M. K. (2005). Using incremental rehearsal to increase fluency of single-digit multiplication facts with children identified as learning disabled in mathematics computation. *Education and Treatment of Children, 28*, 237-249.

Wright, J. www.interventioncentral.org

BARONESS

PURPOSE: Allow students to independently **practice math facts.**

MATERIALS:

- Standard deck of playing cards (Ace = 1 through King)
- Card values are equivalent to value shown on cards
- Face cards' values are: Jack = 11, Queen = 12, King = 13

PROCEDURE:

1. Have students deal a row of 5 cards, face up. The rest of the deck is kept as stock, placed face down

2. As student is looking at the row of 5 cards, have them discard any card numbered 13 (e.g. King) or pairs that total 13

- When 2 cards totaling 13 are removed, they must be taken from columns

3. Starting from the left, deal 5 more cards face up onto the first 5 cards and/or spaces left by cards that have been removed

- Each new row of 5 cards "buries" the cards previously dealt, but new deals should be placed so that the "buried" cards are still visible
- "Buried" cards are unavailable for play until they are released through the removal of overlapping cards
-

4. Discard and deal as before

5. Continue in this manner until the entire deck has been dealt by rows of 5, leaving 2 cards that may be placed to the side and are both available for play

Sources:

Ingham Intermediate School District (Mason, MI) '05-'06 Inservices: *Accountability in the 21st Century, Supporting All Learners Through Collaboration, and Using Your Toolbox to Create a Well-Oiled Machine.*

Usnick, 1991.

TENS

PURPOSE: Allow students to independently **practice math facts.**

MATERIALS:

- Standard deck of playing cards from Ace (= 1) to 10
- Card values are equivalent to value shown on cards
- Include face cards or replace face cards with cards showing geometric shapes

PROCEDURE:

1. Have students deal 9 cards face up in 3 rows of 3 cards (3 x 3). The rest of the deck is kept as stock, placed face down
2. As student is looking at the 9 cards, have them remove pairs of cards whose sum is 10 (e.g. 7 and 3) or pairs of face cards of the same rank (e.g. 2 Jacks)
3. Fill empty spaces by dealing face up from the deck
4. When a 10 is dealt, it blocks further play in that space for the rest of the game
5. Continue until all cards are dealt or no more cards can be paired

Sources:

Ingham Intermediate School District (Mason, MI) '05-'06 In-services: *Accountability in the 21st Century, Supporting All Learners Through Collaboration, and Using Your Toolbox to Create a Well-Oiled Machine.*

Usnick, 1991.

DRAW & FAST DRAW

PURPOSE: The DRAW strategy is used to teach students how to solve **multiplication facts** that are not yet committed to memory. The FAST DRAW strategy helps students make the **transition from pictures to abstract numbers**.

MATERIALS:

- Multiplication worksheets

PROCEDURE: DRAW

1. Explicitly teach students each of the following steps in DRAW
2. **Discover**
 - Discover the sign
 - The student looks at the sign to figure out what operation to perform
3. **Read**
 - Read the problem
 - The student says the problem aloud or to themselves
4. **Answer**
 - Answer, or draw, and check
5. **Write**
 - Write the answer
 - The student writes the answer in the answer space

PROCEDURE: FAST DRAW

1. Explicitly teach students each of the following steps in FAST DRAW
2. **Find**
 - Find what you are solving for
 - Students look for the question in the problem
3. **Ask**
 - Ask yourself “What are the parts of the problem?”
 - Students identify the number of groups and the number of objects in each group
4. **Set**
 - Set up the numbers
 - Students write the 2 numbers in the problem in a vertical format
5. **Tie**
 - Tie down the sign
 - Students add the multiplication sign to the problem

Sources:

Harris, C. A., Miller, S. P., & Mercer, C. D. (1995). Teaching initial multiplication skills to students with disabilities in general education classrooms. *Learning Disabilities Research & Practice, 10*, 180-195.
 Reid, B., & Lienemann, T. <http://www.unl.edu/csi/index.shtml>

TEACHING MULTIPLICATION FACTS

PURPOSE: Increase **fluency** with multiplication facts while simultaneously **decreasing errors**.

MATERIALS:

- Set of flashcards for a set of multiplication facts (for example, multiplication by 3s)
- Construct a worksheet with the same facts randomly arranged
- Timer
- Graph Paper

PROCEDURE:

1. Teacher or tutor presents flashcard to the student while verbally prompting the student with the question (for example, “what is 3 X 3?”)
 - Praise correct answers that occur within 3 seconds of prompting
 - Give student answer if student gives incorrect answer or does not respond within 3 seconds
2. Re-deliver the verbal prompt (for example, “what is 3 X 3?”)
 - Present each card twice
3. Present student with worksheet containing math facts you just presented with flash cards
4. Set timer for 2 minutes
 - Student begins when teacher says “start”
 - Student completes as many problems as they can before timer rings
 - Student works horizontally on paper and does not skip any problems
5. At the end of the 2-minute time period, give student answer key
 - Student circles each error and writes correct response underneath
6. Student calculates the number correct per minute and the number of errors
7. Student records number correct and number of errors on their progress monitoring graph

IS IT WORKING?

- Number of problems correct should increase across days
- Number of errors should decrease across days
- To maximize effects, intervention should be conducted daily

PROMOTING GENERALIZATION

- Conduct sessions with mixed multiplication problems randomly selected from the mastered sets of cards/problems periodically (e.g., once per week)

Sources:

- Bennett, K., & Cavanaugh, R. A. (1998). Effects of immediate self-correction, delayed self-correction, and no correction on the acquisition and maintenance of multiplication facts by a fourth-grade student with learning disabilities. *Journal of Applied Behavior Analysis*, 31, 303-306.
- Rhymer, K. N., Skinner, C. H., Henington, C., D’Reaux, R. A., & Sims, S. (1998). Effects of explicit timing on mathematics problem completion rate in African-American third-grade elementary students. *Journal of Applied Behavior Analysis*, 31, 673-677.
- VanDerHeyden, A., & Witt, J. <http://www.gosbr.net/>

CLASSWIDE MATH INTERVENTION: APPLIED PRACTICE

PURPOSE: To increase **accuracy** and **fluency** for applied problems or word problems.

MATERIALS:

- Worksheets containing math word problems

PROCEDURE: Practice Period (15 minutes)

1. Distribute worksheets and tell students they will be working in pairs
 - Pair students
 - Instruct students to write their name and date on the math sheet
2. Set timer for 2 minutes
 - When timer rings, tell students to stop working
3. Have students trade papers and score
 - Score 1 point for correct equation, 1 point for correct answer, 1 point for labeling answer
4. As teacher gives correct answer, tell students to choral respond each of the 4 problem-solving steps with you. If many students missed a particular step, review the step
5. Have students write correct answer for the problems they missed
3. After each problem, the peer buddy should say “how did you solve the problem” and the student should explain the answer
 - For example “we started with 4 apples and sold 2, so 4 minus 2 equals 2. 2 apples are left. 2 applies is the answer”
4. Peer buddy completes a checklist for each problem as the partner explains the answer, giving a check for each correct step explained (see above steps)
5. Tell students to switch roles – Now other student completes as many problems as possible in 5 minutes with help from their math buddy

PROCEDURE: Evaluation

1. Pass out worksheet while students are finishing their second set of practice problems
2. Set timer for 2 minutes
 - When timer rings, tell students to stop working

Math Problem-Solving: Combining Cognitive & Metacognitive Strategies in a 7-Step Process

PURPOSE: Provide **explicit cognitive** and **metacognitive** series of steps to help **analyze and solve a math problem**. The metacognitive strategy sequence (see below) is repeated across all 7 problem-solving steps of the cognitive strategy.

MATERIALS:

- Worksheets containing math word problems

PROCEDURE: Cognitive Strategy

1. Reading the Problem
 - Student reads problem carefully
 - Student notes and attempts to clear up any areas of confusion (e.g. unknown vocab terms)
2. Paraphrasing the Problem
 - Student restates the problem in their own words
3. “Drawing” the Problem
 - Student creates visual representation of the word problem
4. Creating a Plan to Solve the Problem
 - Student decides best way to solve problem
 - Student creates a plan to carry out
5. Predicting/Estimating the Answer
 - Student estimates or predicts answer by computing a quick approximation (e.g. shortcuts, rounding)
6. Computing the Answer
 - Student follows plan they developed to compute answer
7. Checking the Answer
 - Student methodically checks calculations for each step of problem
 - Student compares actual answer to estimated answer to ensure agreement

PROCEDURE: Metacognitive Strategy

1. Say – Student self-instructs by “saying” purpose of the step
2. Ask – Student self-questions by “asking” what they intend to do to complete the step
3. Check – Student “checks” the successful completion of the step

Sources:

- Burns, M. K., VanDerHeyden, A. M., & Boice, C. H. (2008). Best practices in intensive academic interventions. In A. Thomas & J. Grimes (Eds.), *Best practices in school psychology V* (pp.1151-1162). Bethesda, MD: National Association of School Psychologists.
- Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. *Journal of Learning Disabilities, 25*, 230-248.
- Montague, M., & Dietz, S. (2009). Evaluating the evidence base for cognitive strategy instruction and mathematical problem solving. *Exceptional Children, 75*, 285-302.
- Wright, J. www.interventioncentral.org

APPLIED MATH PROBLEMS: USING QUESTION-ANSWER RELATIONSHIPS (QAR) TO INTERPRET MATH GRAPHICS

PURPOSE: An instructional strategy to help students increase **graph interpretation skills**.

MATERIALS:

- Worksheets containing graphic math problems

PROCEDURE:

1. Distinguishing Among Different Kinds of Graphics

- Teach students to discriminate between 5 common types of math graphs
 - Table – grid with information contained in cells
 - Chart – boxes with possible connecting lines or arrows
 - Picture – figure with labels
 - Line Graph
 - Bar Graph
- Students note differences between the types of graphs and teacher records the observations on a wall chart
- Students are shown examples of graphs and are asked to identify the general type of graph it is
- As homework, students are assigned to go on a “graphic hunt” locating graphs in magazines, newspapers, labeling them, and bringing them to class to review

2. Interpreting Information in Graphics

- Teacher teaches each of the graphs in separate sessions
- Presentation sequence is ordered so students begin with examples of the most concrete graphs and move toward more abstract
- Level of increasing difficulty is:
 - Pictures → Tables → Bar Graphs → Charts → Line Graphs
- Stronger students are paired with less strong students when practicing how to interpret information contained in various graphs
- Student pairs explore graphs and discuss questions such as:
 - What information does this graph represent?
 - What are strengths and weaknesses of this type of graph for presenting the data?
- Student pairs record their findings and share them with the large group at the end of the session

3. Linking the Use of Question-Answer Relationships (QARs) to Graphics

- Prior to this lesson, teacher prepares data questions and answers. Each question and answer is paired with a graph that contains information essential for finding the answer
- Students are given 4 index cards with titles and descriptions of each of the 4 QAR questions:
 - RIGHT THERE
 - THINK AND SEARCH
 - AUTHOR AND YOU
 - ON MY OWN
- Working in small groups then individually, students read each graph and question, and “verify” the provided answer is correct
- Students identify the type of question being asked in each problem using their QAR cards

4. Using Question-Answer Relationships (QARs) Independently to Interpret Math Graphs

- Students are now ready to use QAR strategy independently
- Students are given a laminated reference card with 6 steps to follow whenever they attempt to solve a graph problem. The 6 steps include:
 - Read the Question
 - Review the Graph
 - Reread the Question
 - Choose a Question-Answer Relationship that matches the question in the problem
 - Answer the Question
 - Locate the answer derived from the graph in the choices offered

Sources:

Mesmer, H. A. E., & Hutchins, E. J. (2002). Using QARs with charts and graphs. *The Reading Teacher*, 56, 21-27.

Raphael, T. (1982). Question-answering strategies for children. *The Reading Teacher*, 36, 186-190.

Raphael, T. (1986). Teaching question answer relationships, revised. *The Reading Teacher*, 39, 516-522.

Wright, J. www.interventioncentral.org